

```

> restart: with(LinearAlgebra): with(Groebner):

m := 7;

twist := - 5;

p := 5;

dR := 3*(m-0) + twist:
dS := 3*(m-2) + twist:
dU := 3*(m-4) + twist:
dV := 3*(m-6) + twist:
dW := 3*(m-8) + twist:

##

AA := 2*T^2 + 1*X^2 + 1*Y^2;
BB := 1*T^2 + 2*X^2 + 1*Y^2;
CC := 1*T^2 + 1*X^2 + 2*Y^2;

#

AT := diff(AA,T): BT := diff(BB,T): CT := diff(CC,T):
AX := diff(AA,X): BX := diff(BB,X): CX := diff(CC,X):
AY := diff(AA,Y): BY := diff(BB,Y): CY := diff(CC,Y):

#

ATT := diff(AA,T,T): ATX := diff(AA,T,X): ATY := diff(AA,T,Y):
AXX := diff(AA,X,X): AXY := diff(AA,X,Y): AYY := diff(AA,Y,Y):
BTT := diff(BB,T,T): BTX := diff(BB,T,X): BTY := diff(BB,T,Y):
BXX := diff(BB,X,X): BXY := diff(BB,X,Y): BYY := diff(BB,Y,Y):
CTT := diff(CC,T,T): CTX := diff(CC,T,X): CTY := diff(CC,T,Y):
CXX := diff(CC,X,X): CXY := diff(CC,X,Y): CYY := diff(CC,Y,Y):

#

MatDD := Matrix([[AT,BT,CT],
                  [AX,BX,CX],
                  [AY,BY,CY]]):

DD := factor(Determinant(MatDD));

##

```

Ap := AT*T1 + AX*X1 + AY*Y1:

Bp := BT*T1 + BX*X1 + BY*Y1:

Cp := CT*T1 + CX*X1 + CY*Y1:

App := + AT*T2 + AX*X2 + AY*Y2

**+ ATT*T1*T1 + 2*ATX*T1*X1 + 2*ATY*T1*Y1 + AXX*X1*X1 + 2*
AXY*X1*Y1 + AYY*Y1*Y1:**

Bpp := + BT*T2 + BX*X2 + BY*Y2

**+ BTT*T1*T1 + 2*BTX*T1*X1 + 2*BTY*T1*Y1 + BXX*X1*X1 + 2*
BXY*X1*Y1 + BYY*Y1*Y1:**

Cpp := + CT*T2 + CX*X2 + CY*Y2

**+ CTT*T1*T1 + 2*CTX*T1*X1 + 2*CTY*T1*Y1 + CXX*X1*X1 + 2*
CXY*X1*Y1 + CYY*Y1*Y1:**

##

logACp := Ap/AA - Cp/CC:

logBCp := Bp/BB - Cp/CC:

logACpp := App/AA - Ap^2/AA^2 - (Cpp/CC - Cp^2/CC^2):

logBCpp := Bpp/BB - Bp^2/BB^2 - (Cpp/CC - Cp^2/CC^2):

##

**Carte_xy := {T =1, X =x, Y =y,
T1=0, X1=x1, Y1=y1,
T2=0, X2=x2, Y2=y2}:**

xyAA := subs(Carte_xy, AA):

xyBB := subs(Carte_xy, BB):

xyCC := subs(Carte_xy, CC):

xyDD := subs(Carte_xy, DD):

xylogACp := subs(Carte_xy, logACp):

xylogBCp := subs(Carte_xy, logBCp):

xylogACpp := subs(Carte_xy, logACpp):

xylogBCpp := subs(Carte_xy, logBCpp):

##

**Carte_tx := {T =t, X =x, Y =1,
T1=t1, X1=x1, Y1=0,
T2=t2, X2=x2, Y2=0}:**

txAA := subs(Carte_tx, AA):

```

txBB := subs(Carte_tx, BB):
txCC := subs(Carte_tx, CC):
txDD := subs(Carte_tx, DD):

```

```

txlogACp := subs(Carte_tx, logACp):
txlogBCp := subs(Carte_tx, logBCp):

```

```

txlogACpp := subs(Carte_tx, logACpp):
txlogBCpp := subs(Carte_tx, logBCpp):

```

```

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```

Jet_ABC := + (1/(T*X*Y)^(m-0))
              *
              add(R[m-0-j,j] * logACp^(m-0-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^0, j=0..m-0)

+ (1/(T*X*Y)^(m-2))
  *
  add(S[m-3-j,j] * logACp^(m-3-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^1, j=0..m-3)

+ (1/(T*X*Y)^(m-4))
  *
  add(U[m-6-j,j] * logACp^(m-6-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^2, j=0..m-6)

+ (1/(T*X*Y)^(m-6))
  *
  add(V[m-9-j,j] * logACp^(m-9-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^3, j=0..m-9)

+ (1/(T*X*Y)^(m-8))
  *
  add(W[m-12-j,j] * logACp^(m-12-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^4, j=0..m-12):

```

```

##

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```

xyJet_ABC := factor(+ (1/(x*y)^(m-0))
                      *
                      add(Rxy[m-0-j,j] * xylogACp^(m-0-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^0, j=0..m-0)

+ (1/(x*y)^(m-2))
  *
  add(Sxy[m-3-j,j] * xylogACp^(m-3-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^1, j=0..m-3)

```

```

+ (1/(x*y)^(m-4))
*
add(Uxy[m-6-j,j] * xylogACp^(m-6-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^2, j=0..m-6)

+ (1/(x*y)^(m-6))
*
add(Vxy[m-9-j,j] * xylogACp^(m-9-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^3, j=0..m-9)

+ (1/(x*y)^(m-8))
*
add(Wxy[m-12-j,j] * xylogACp^(m-12-j) *
xylogBCp^j * (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^4, j=0..
m-12)):

```

#

```

txJet_ABC := + (1/(t*x)^(m-0))
*
add(Rtx[m-0-j,j] * txlogACp^(m-0-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^0, j=0..m-0)

+ (1/(t*x)^(m-2))
*
add(Stx[m-3-j,j] * txlogACp^(m-3-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^1, j=0..m-3)

+ (1/(t*x)^(m-4))
*
add(Utx[m-6-j,j] * txlogACp^(m-6-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^2, j=0..m-6)

+ (1/(t*x)^(m-6))
*
add(Vtx[m-9-j,j] * txlogACp^(m-9-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^3, j=0..m-9)

+ (1/(t*x)^(m-8))
*
add(Wtx[m-12-j,j] * txlogACp^(m-12-j) *
txlogBCp^j * (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^4, j=0..
m-12):

```

##

```

NFxy := proc(P,j,i)

```

```
expand(maylor(P,x,j) + maylor(P,y,i) - maylor(maylor(P,y,i),
x,j)):

```

```
end proc:

```

```
#

```

```
NFtx := proc(P,j,i)

```

```
expand(maylor(P,t,j) + maylor(P,x,i) - maylor(maylor(P,x,i),
t,j)):

```

```
end proc:

```

$m:=7$

$twist:=-5$

$p:=5$

$AA:=2T^2+X^2+Y^2$

$BB:=T^2+2X^2+Y^2$

$CC:=T^2+X^2+2Y^2$

$DD:=32TXY$

(1)

```
> xyJet := maylor(

```

```
factor((x*y*xyAA*xyBB*xyCC)^m * (subs(y2=(xyW+x2*y1)/x1,
xyJet_ABC)))

```

```
, [x1,y1,xyW], 100):

```

```
#

```

```
txJet := maylor(

```

```
factor((t*x*txAA*txBB*txCC)^m * (subs(x2=(txW+t2*x1)/t1,
txJet_ABC)))

```

```
, [t1,x1,txW], 100):

```

```

> printlevel := 2:
  for k from 0 to m/3 do
    for j from 0 to m-3*k do

      xyEEEEq[m-j-3*k,j,k] := factor(coeftayl(xyJet, [x1,y1,xyW]=[0,0,
      0], [m-j-3*k,j,k])):

    od: od:

  #

  printlevel := 2:
  for k from 0 to m/3 do
    for j from 0 to m-3*k do

      txEEEEq[m-j-3*k,j,k] := factor(coeftayl(txJet, [t1,x1,txW]=[0,0,
      0], [m-j-3*k,j,k])):

    od: od:

```

```

> printlevel := 2:
  for k from 0 to m/3 do
    for j from 0 to m-3*k do

      xyEEEEq[m-j-3*k,j,k] := mtaylor(

          factor(
              (1/(x^(m-j-3*k)*y^j*(x*y)^(3*k))) *
              (1/((xyAA*xyBB*xyCC)^(2*k)))
              *
              xyEEEEq[m-j-3*k,j,k])

          , [x,y], 1000):

    od: od:

  #

  printlevel := 2:
  for k from 0 to m/3 do
    for j from 0 to m-3*k do

      txEEEq[m-j-3*k,j,k] := mtaylor(

```

```

factor(
    (1/(t^(m-j-3*k)*x^j*(t*x)^(3*k))) *
    (1/((txAA*txBB*txCC)^(2*k)))
    *
    txEEEEq[m-j-3*k,j,k])
, [t,x], 1000):
od: od:

```

```

> printlevel := 2:
for k from 0 to m/3 do
for j from 0 to m-3*k do

xyEEq[m-j-3*k,j,k] := NFxy(xyEEEq[m-j-3*k,j,k], j,m-j-3*k) mod p:

od: od:

#

printlevel := 2:
for k from 0 to m/3 do
for j from 0 to m-3*k do

txEEq[m-j-3*k,j,k] := NFtx(txEEEq[m-j-3*k,j,k], j,m-j-3*k) mod p:

od: od:

```

```

> for j from 0 to m-0 do
Rxy[m-0-j,j] := add(add(cR[m-0-j,j,dR-q-r,q,r]*1^(dR-q-r)*x^q*
y^r, q=0..dR-r), r=0..dR):
od:

for j from 0 to m-3 do
Sxy[m-3-j,j] := add(add(cS[m-3-j,j,dS-q-r,q,r]*1^(dS-q-r)*x^q*
y^r, q=0..dS-r), r=0..dS):
od:

for j from 0 to m-6 do
Uxy[m-6-j,j] := add(add(cU[m-6-j,j,dU-q-r,q,r]*1^(dU-q-r)*x^q*

```

```
y^r, q=0..dU-r), r=0..dU):  
od:
```

```
for j from 0 to m-9 do  
Vxy[m-9-j,j] := add(add(cV[m-9-j,j,dV-q-r,q,r]*1^(dV-q-r)*x^q*  
y^r, q=0..dV-r), r=0..dV):  
od:
```

```
for j from 0 to m-12 do  
Wxy[m-12-j,j] := add(add(cW[m-12-j,j,dW-q-r,q,r]*1^(dW-q-r)*x^q*  
y^r, q=0..dW-r), r=0..dW):  
od:
```

```
#
```

```
for j from 0 to m-0 do  
Rtx[m-0-j,j] := add(add(cR[m-0-j,j,dR-q-r,q,r]*t^(dR-q-r)*x^q*  
1^r, q=0..dR-r), r=0..dR):  
od:
```

```
for j from 0 to m-3 do  
Stx[m-3-j,j] := add(add(cS[m-3-j,j,dS-q-r,q,r]*t^(dS-q-r)*x^q*  
1^r, q=0..dS-r), r=0..dS):  
od:
```

```
for j from 0 to m-6 do  
Utx[m-6-j,j] := add(add(cU[m-6-j,j,dU-q-r,q,r]*t^(dU-q-r)*x^q*  
1^r, q=0..dU-r), r=0..dU):  
od:
```

```
for j from 0 to m-9 do  
Vtx[m-9-j,j] := add(add(cV[m-9-j,j,dV-q-r,q,r]*t^(dV-q-r)*x^q*  
1^r, q=0..dV-r), r=0..dV):  
od:
```

```
for j from 0 to m-12 do  
Wtx[m-12-j,j] := add(add(cW[m-12-j,j,dW-q-r,q,r]*t^(dW-q-r)*x^q*  
1^r, q=0..dW-r), r=0..dW):  
od:
```

```
> printlevel := 2:  
for k from 0 to m/3 do  
for j from 0 to m-3*k do
```

```
xyEq[m-j-3*k,j,k] := NFxy(xyEEq[m-j-3*k,j,k], j,m-j-3*k):
```

```
od: od:
```

```
#
```

```
printlevel := 2:
```

```
for k from 0 to m/3 do
```

```
for j from 0 to m-3*k do
```

```
txEq[m-j-3*k,j,k] := NFtx(txEEq[m-j-3*k,j,k], j,m-j-3*k):
```

```
od: od:
```

```
> printlevel := 2:
for k from 0 to m/3 do
for j from 0 to m-3*k do

xyEquations[m-j-3*k,j,k] := {coeffs(xyEq[m-j-3*k,j,k], [x,y])}:
od: od:
#
```

```
printlevel := 2:
```

```
for k from 0 to m/3 do
```

```
for j from 0 to m-3*k do
```

```
txEquations[m-j-3*k,j,k] := {coeffs(txEq[m-j-3*k,j,k], [t,x])}:
od: od:
```

```
> xyEquations_Add := {}:
```

```
for k from 0 to m/3 do
```

```
for j from 0 to m-3*k do
```

```

xyEquations_Add := xyEquations_Add union xyEquations[m-j-3*k,j,k]
:
od: od:
Equations_xy := xyEquations_Add:
#
txEquations_Add := {}:
for k from 0 to m/3 do
for j from 0 to m-3*k do

txEquations_Add := txEquations_Add union txEquations[m-j-3*k,j,k]
:
od: od:
Equations_tx := txEquations_Add:

```

```

> Equations := Equations_xy union Equations_tx mod p:
Nombre_Equations := nops(Equations);

```

Nombre_Equations := 4218

(2)

```

> Variables := {seq(seq(seq(cR[m-0-j,j,dR-q-r,q,r], q=0..dR-
r), r=0..dR), j=0..m-0)}
union {seq(seq(seq(cS[m-3-j,j,dS-q-r,q,r], q=0..dS-
r), r=0..dS), j=0..m-3)}
union {seq(seq(seq(cU[m-6-j,j,dU-q-r,q,r], q=0..dU-
r), r=0..dU), j=0..m-6)}
union {seq(seq(seq(cV[m-9-j,j,dV-q-r,q,r], q=0..dV-
r), r=0..dV), j=0..m-9)}
union {seq(seq(seq(cW[m-12-j,j,dW-q-r,q,r], q=0..dW-
r), r=0..dW), j=0..m-12)}:
Nombre_Variables := nops(Variables);

```

```
[> Solutions := msolve(Equations,p):
```

```
[> Verification := seq(op(2,op(i,Solutions)),i=1..nops(Solutions));
indets({Verification}):
nops(indets({Verification})):
```

```
Verification:= 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

(4)

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```

od;

for j from 0 to m-3 do
Voir_S[m-3-j,j] := add(add(cS[m-3-j,j,dS-q-r,q,r]*T^(dS-q-r)*X^q*
Y^r, q=0..dS-r), r=0..dS):
od;

for j from 0 to m-6 do
Voir_U[m-6-j,j] := add(add(cU[m-6-j,j,dU-q-r,q,r]*T^(dU-q-r)*X^q*
Y^r, q=0..dU-r), r=0..dU):
od;

for j from 0 to m-9 do
Voir_V[m-9-j,j] := add(add(cV[m-9-j,j,dV-q-r,q,r]*T^(dV-q-r)*X^q*
Y^r, q=0..dV-r), r=0..dV):
od;

for j from 0 to m-12 do
Voir_W[m-12-j,j] := add(add(cW[m-12-j,j,dW-q-r,q,r]*T^(dW-q-r)*
X^q*Y^r, q=0..dW-r), r=0..dW):
od;

```

$$Voir_{R_{7,0}} := 0$$

$$Voir_{R_{6,1}} := 0$$

$$Voir_{R_{5,2}} := 0$$

$$Voir_{R_{4,3}} := 0$$

$$Voir_{R_{3,4}} := 0$$

$$Voir_{R_{2,5}} := 0$$

$$Voir_{R_{1,6}} := 0$$

$$Voir_{R_{0,7}} := 0$$

$$Voir_{S_{4,0}} := 0$$

$$Voir_{S_{3,1}} := 0$$

$$Voir_{S_{2,2}} := 0$$

$$Voir_{S_{1,3}} := 0$$

$$Voir_{S_{0,4}} := 0$$

$$Voir_{U_{1,0}} := 0$$

$$Voir_{U_{0,1}} := 0$$

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